Cameron–Martin formula for σ-finite measure unifying Brownian penalisations

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Quasi-invariance under translation is established for the σ-finite measure unifying Brownian penalisations, which has been introduced by Najnudel, Roynette and Yor ([2] and [3]). For this purpose, the theory of Wiener integrals for centered Bessel processes, due to Funaki, Hariya and Yor ([1]), plays a key role.

Let $\{(X_t), (\mathcal{F}_t), W\}$ denote the canonical representation of 1-dimensional Brownian motion starting from the origin. The measure mentioned above is defined as follows:

$$\mathscr{W} = \int_0^\infty \frac{\mathrm{d}u}{\sqrt{2\pi u}} \left(\Pi^{(u)} \bullet R\right) \tag{1}$$

where $\Pi^{(u)}$ stands for the law of Brownian bridge from 0 to 0 of length u, and R for the law of symmetrized 3-dimensional Bessel process. Now the main theorem is stated as follows.

Theorem 1 ([5],[4]). *Suppose that $h_t = \int_0^t f(s)\mathrm{d}s$ with $f \in L^2(\mathrm{d}s) \cap L^1(\mathrm{d}s)$. Then it holds that*

$$\mathscr{W}[F(X+h)] = \mathscr{W}[F(X)\mathcal{E}(h;X)] \tag{2}$$

for any non-negative $\mathcal{F}_\infty$-measurable functional F where

$$\mathcal{E}(h;X) = \exp\left(\int_0^\infty f(s)\mathrm{d}X_s - \frac{1}{2}\int_0^\infty f(s)^2\mathrm{d}s\right). \tag{3}$$

References

[1] T. Funaki, Y. Hariya, and M. Yor. Wiener integrals for centered Bessel and related processes. II. *ALEA Lat. Am. J. Probab. Math. Stat.*, 1:225–240 (electronic), 2006.

[2] J. Najnudel, B. Roynette, and M. Yor. A remarkable σ-finite measure on $\mathcal{C}(\mathbf{R}_+, \mathbf{R})$ related to many Brownian penalisations. *C. R. Math. Acad. Sci. Paris*, 345(8):459–466, 2007.

[3] J. Najnudel, B. Roynette, and M. Yor. *A global view of Brownian penalisations*, volume 19 of *MSJ Memoirs*. Mathematical Society of Japan, Tokyo, 2009.

[4] K. Yano. Cameron–Martin formula for the σ-finite measure unifying Brownian penalisations. *preprint, arXiv:0909.5132*, 2009.

[5] K. Yano. Wiener integral for the coordinate process under the σ-finite measure unifying brownian penalisations. *preprint, arXiv:0909.5130*, 2009.