Markov Property of Dyson's Model with an Infinite Number of Particles

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(*joint work with Makoto Katori, Chuo University*)

We consider the process $\Xi(t) = \sum_{j=1}^{N} \delta_{X_j(t)}$ with the SDEs

$$dX_j(t) = dB_j(t) + \sum_{1 \le k \le N, k \ne j} \frac{dt}{X_j(t) - X_k(t)}, \quad 1 \le j \le N, \quad t \in [0, \infty),$$

where $B_j(t)$'s are independent one-dimensional standard Brownian motions. The state space of the process is the space of nonnegative integer-valued Radon measures $\mathfrak{M}$ on $\mathbb{R}$, which is a Polish space with the vague topology. In [2] we gave sufficient conditions for an initial configuration $\xi = \sum_{j=1}^{\infty} \delta_{x_j} \in \mathfrak{M}$ such that the process $(\mathbb{P}^{\xi \cap [-L,L]}, \Xi(t))$ converges to an $\mathfrak{M}$-valued process, say $(\mathbb{P}^{\xi}, \Xi(t))$, as $L \to \infty$, in the sense of finite dimensional distributions, where $\xi \cap [-L, L]$ is the restriction of ξ on $[-L, L]$, i.e. $\xi \cap [-L, L] = \sum_{j: x_j \in [-L,L]} \delta_{x_j}$.

In this talk we discuss the following:

1. Thightness of the sequence of the processes $(\mathbb{P}^{\xi \cap [-L,L]}, \Xi(t))$, $L \in \mathbb{N}$.
2. Markov property of the limit process $(\mathbb{P}^{\xi}, \Xi(t))$.

References

[1] Katori, M., Tanemura, H.: Noncolliding Brownian motion and determinantal processes. J. Stat. Phys. **129**, 1233-1277 (2007)

[2] Katori, M., Tanemura, H.: Non-equilibrium dynamics of Dyson's model with an infinite number of particles. Commun. Math. Phys. DOI:10.1007/s00220-009-0912-3; arXiv:math.PR/0812.4108.

[3] Katori, M., Tanemura, H.: Zeros of Airy function and relaxation process. J. Stat. Phys. DOI:10.1007/s10955-009-9829-7; arXiv:math.PR/0906.3666.