Exact value of the resistance exponent for four dimensional random walk trace

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Abstract

Let S be a simple random walk starting at the origin in $\mathbb{Z}^4$. We consider $\mathcal{G} = S[0,\infty)$ to be a random subgraph of the integer lattice and assume that a resistance of unit 1 is put on each edge of the graph $\mathcal{G}$. Let R_n be the effective resistance between the origin and S_n. We derive the exact value of the resistance exponent; more precisely, we prove that $n^{-1}E(R_n) \approx (\log n)^{-\frac{1}{2}}$. Furthermore, we derive the precise exponent for the heat kernel of a random walk on $\mathcal{G}$ at the quenched level. These results give the answer to the problem raised by Burdzy and Lawler (1990) in four dimensions.

References

[1] Burdzy, K.; Lawler, G. F. : Rigorous exponent inequalities for random walks. Journal of Physics A: Mathematical and General, Volume 23, Issue 1, pp. L23-L28 (1990).