On the maximum of Dyson Brownian motion

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Dyson's Brownian motion is a stochastric process described by the SDE,

$$dX_i = dB_i + \frac{\beta}{2} \sum_{\substack{1\le j\le m \\ j\neq i}} \frac{dt}{X_i - X_j}, \quad 1 \le i \le m$$

where $B_i, 1 \le i \le m$ are independent 1D Brownian motions. In this talk, we only consider the case $\beta = 2$. It is known that the initial conditions can be taken to be $X_i(0) = 0, 1 \le i \le m$ and that the process satisfies $X_1(t) < X_2(t) < \cdots < X_m(t), \quad \forall t > 0$. This can be interpreted as a system of m Brownian particles conditioned to never collide with each other.

When $m = 1$, this is simply a single Brownian motion. It is well known that $\max_{0\le s\le t} B(s)$ has the same distribution as the reflected BM.

One can also define Dyson's Brownian motion of type C, $X^{(C)}$, as a system of m Brownian particles conditioned to never collide with each other or the wall and that of type D, $X^{(D)}$, as as a system of m reflected Brownian particles conditioned to never collide with each other. Then we show

Theorem. Let X and $X^{(C)}, X^{(D)}$ start from the origin. Then for each fixed $t \ge 0$, one has

$$\sup_{0\le s\le t} X_n(s) \stackrel{d}{=} \begin{cases} X_m^{(C)}(t), & \text{for } n = 2m, \\ X_m^{(D)}(t), & \text{for } n = 2m-1. \end{cases}$$

This is a multi-dimensional generalization of the above mentioned relation between maximum of BM and BM with a reflecting boundary.

Originally this relation was anticipated from a consideration of the diffucion scaling limit in the TASEP with two speeds but can be shown without refering to TASEP. The idea of the proof is to introduce two new new processesZ, Y and show

$$\max X = \max Z = Y = X^{(C,D)}.$$

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References

[1] A. Borodin, P. L. Ferrari, M. Prähofer, T. Sasamoto, J. Warren, arXiv:0905.3989.
[2] A. Borodin, P. L. Ferrari, T. Sasamoto, arXiv:0904.4655, to appear in J. Stat. Phys.